Lab 17

Two-Dimensional Electric Field and Potential

A. Purpose

To investigate the electric field and potential distribution on a two-dimensional (2D) carbon-coated conductive plate.

B. Introduction

The concept of fields is generally used to describe the interactions between the source and the test particle in the space. For example, the force F exerted on a test charge q_0 is used to define the electric field in the space:

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \tag{1}$$

Then alternatively, one can say that the force exerted on the charge is $\mathbf{F} = q_0\mathbf{E}$. Note that the electric field is a vector with the same direction as the force exerted on the test charge. In 1852, Michael Faraday proposed the idea of lines of force to bring the distribution of abstract fields into concrete and seeable pictures. (See Fig. 1)

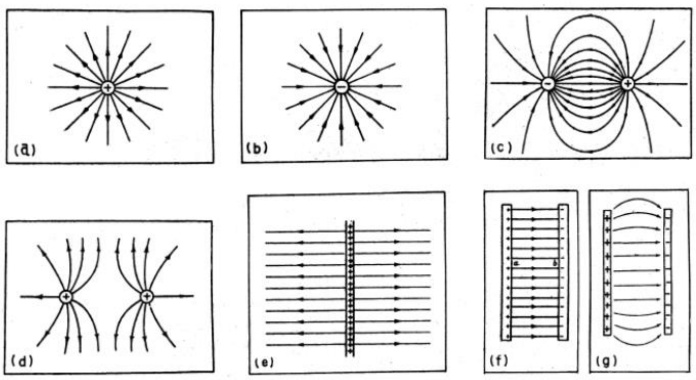


Fig. 1. Lines of electric fields created by different charge distribution: (a) positive charge (b) negative charge (c) a pair of positive and negative charges (d) two positive charges (e) an infinite plate with positive charge (f) two parallel infinite plates with opposite charge (g) two parallel finite plates with opposite charge

Electric field lines tell the strength and the direction of the electric field in the space.

(1) Direction of E-field: the tangential direction of any point on the electric field lines is the direction of the E-field. For example, the direction of the E-field points outward from the positive charge as shown in Fig. 1(a) or inward to the negative charge as shown in Fig. 1(b).

(2) Strength of E-field: the denser the field lines, the stronger the E-field. An uniform E-field is indicated by several parallel field lines, as shown in Fig. 1(e) and 1(f).

Also, note that any ***two field lines never intersect with each other***.

How the point charge interacts with the electric field in the space can also be described by a scalar electric potential V. To move a point charge q from point A to point B in an electric field, the work W_{AB} exerted on the charge by the electric field is used to define the electric potential difference between the two points.

$$V_{BA} = V_B - V_A = \frac{W_{AB}}{q}$$

The potential difference is relative. For example, suppose point A is at infinity where the electric field is negligible. In that case, point A's potential is zero, and the potential difference between the two points is defined as point B's (absolute) potential. If we connect all the same potential points in space, the equipotential surfaces are formed. There are some properties of the equipotential surfaces.

(1) No work is required to move a charge anywhere along the equipotential surfaces.
(2) The electric field lines are always perpendicular to the equipotential surfaces.
(3) There is no electric field in a conductor; potentials inside the conductor are the same.
(4) The electric field lines are always perpendicular to the surface of a conductor.

The relationship between equipotential surfaces and electric field lines is shown in Fig. 2.

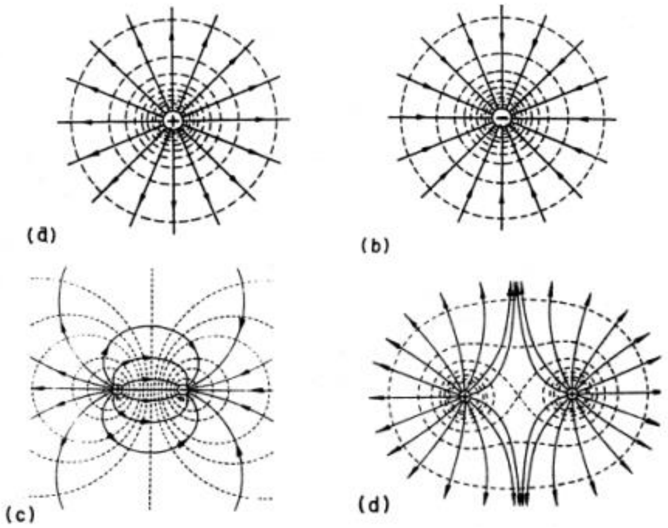


Fig. 2. Equipotential lines (dotted lines) and electric field lines (solid lines) under different charge distributions. (a) positive charge (b) negative charge (c) positive-negative charge pair (d) two positive charges

How to draw electric field lines in this lab?

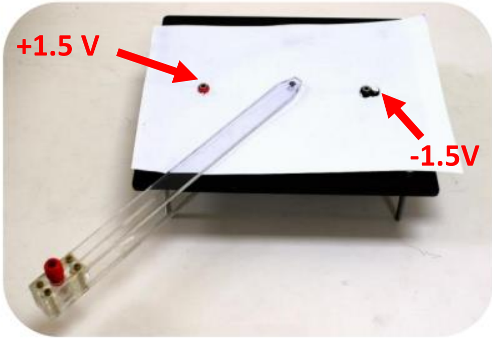
Before going into detail, it's important to mention that there is no charge on the 2D plates in this lab. Here, we set the potentials of two given points on the plate, resulting in the plate's potential distribution. Equivalently, the electric field distribution is formed. Now, to draw the electric lines, recall the relation between electric field and the electric potential:

$$\mathbf{E} = -\nabla V$$

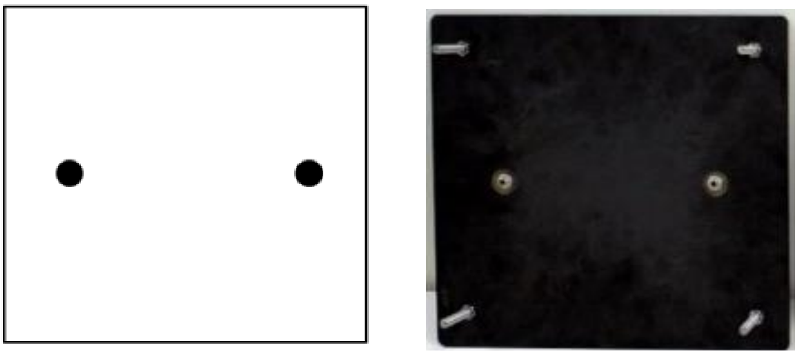
Start from point A on the plate and draw a circle with a small radius r. Then, based on the definition of the gradient, the electric field at A is directed to the point on the circle that has the maximum potential difference from A.

C. Apparatus

Power supply



U-type probe on a carbon-coated conductive plate



Carbon-coated conductive plates with point-point electrodes

D. Procedures

1. Pre-lab assignments (hand in before the experiment)
   - (0) See the following link for a brief introduction of this laboratory: https://youtu.be/JGwSjQukZFY
   - (1) Make sure you know how to simulate the experiment by Matlab. (see Appendix)
   - (2) Make a flowchart of this experiment and answer the questions below.
   - (3) Consider a 2D circular plate with a circular positive electrode $(+V)$ at the center. Find the potential distribution $V(r)$ on the plate. (Take infinity as the zero point of the potential)
   - (4) Prove that for a 3D space, given a positive spherical electrode $(+V)$ of radius R at $+\mathbf{d}$ and a negative spherical electrode $(-V)$ of the same radius at $-\mathbf{d}$, the potential of a point at $\mathbf{r}$ outside the electrodes is

$$V_{3D} = C_{3D}\left(\frac{1}{|\mathbf{r}-\mathbf{d}|} - \frac{1}{|\mathbf{r}+\mathbf{d}|}\right)$$

where C_{3D} is a constant to be determined by the boundary condition.

(5) Following 4, prove that for the 2D case, the potential is

$$V_{2D} = C_{2D}\left(\ln|\mathbf{r}+\mathbf{d}| - \ln|\mathbf{r}-\mathbf{d}|\right)$$

where C_{2D} is a constant to be determined by the boundary condition.

(6) Explain the principle of drawing electric field lines in this laboratory. (Hint: recall the *geometric definition of gradient*)

2. In-lab activities (Don't forget to take a photo of the back of the plate)

(1) Potentials along the midline

(i) Place the given recording paper on the plate and use the banana cables to connect the two electrodes of the plates with the power supply. Set the voltage difference to be 3 V. (+1.5V for the positive electrode and -1.5V for the negative.)

(ii) Connect the U-type probe with the multimeter, and connect COMs between the multimeter and the power supply so that the potential measured by the probe has the same zero point as the applied potentials.

(iii) Use the multimeter with the probe to measure the potential versus the distance along the midline connecting the two electrodes. The distance between two consecutive points should be about 1 cm or shorter.

(iv) Use the data you obtained to prove that the potential distribution should be described by the 2D formula instead of the 3D one on the plate.

(a) Use the boundary condition to determine the 2D and 3D formulas.(pre-lab)

(b) Plot the potential versus the position described by the 2D and the 3D formulas and the data you obtained.

(2) Equipotential lines

Following exp 1, use the multimeter with the probe to find the same potential points to form equipotential lines. The potentials of the lines are +1.2 V to -1.2 V, with the potential difference between two consecutive lines being 0.3 V. For each line, at least 15 points of the same potential are needed to form a smooth line. **The data points you obtained should be evenly spread within the boundary of the paper.**

(3) Electric field lines

Following exp 2, centered at a point around the electrode, draw an arc of radius $r = 1.5$ cm. Find the point on the arc that has the maximum potential difference from the original point. Repeat the process with the point you just found being the arc center until arriving at the other electrode. ***Five electric field lines are needed***, and the black points in Fig. 3 show their starting points.

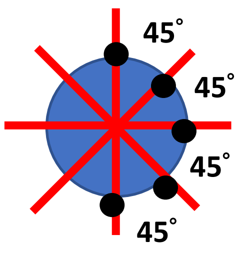


Fig. 3. Starting points of the electric field lines

(4) Matlab Simulation

Measure the radius of the electrodes and the distance between the electrodes. Then, use Matlab to simulate the theoretical equipotential lines and the electric field lines. Finally, compare the simulation with experimental results by dotting the experimental data in the Matlab simulation.

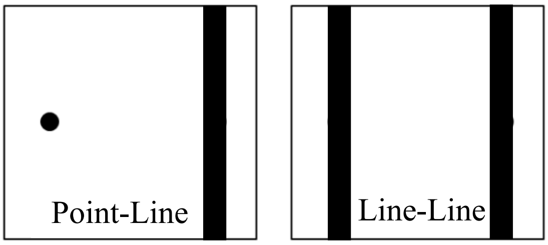
3. Post-lab report

(1) Recopy and organize your data from the in-lab tables in a neat and more readable form.

(2) Analyze the data you obtained in the lab and answer the given questions

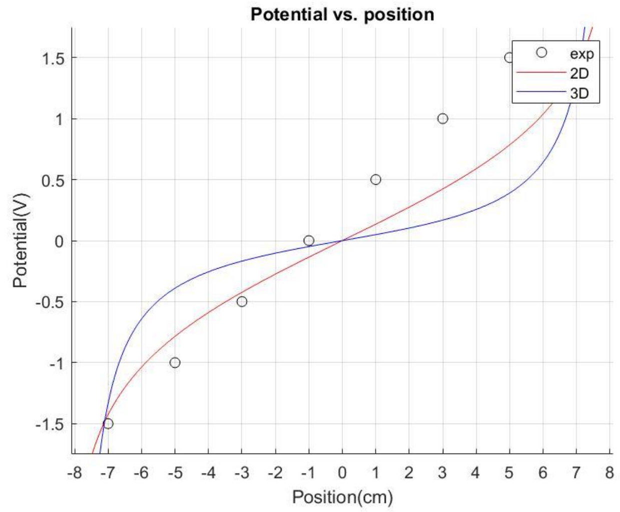
E. Questions

1. While drawing electric field lines, you are asked to draw an arc of radius $r = 1.5\ \mathrm{cm}$. If one instead draws an arc of radius $r = 3\ \mathrm{cm}$, or even larger, what is the difference? Explain by comparing the simulation results obtained by different radii.
2. Would the finite dimension of the plate affect the experimental results? Explain by comparing the experimental results with your simulation.
3. **(Optional)** Use Matlab to simulate equipotential and electric field lines on the following infinite conductive plates, where the black color shows the position of the electrodes.



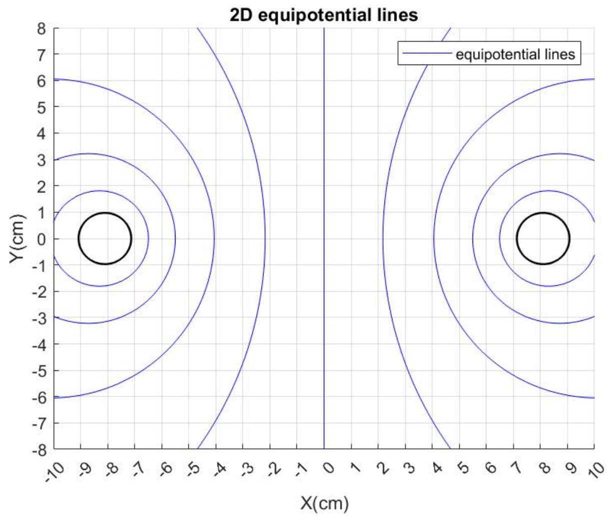
Appendix: Matlab codes examples

```
C2D = -0.54452;
C3D = 1.58459;

x = linspace(-8.1, 8.1, 1000);
y2D = C2D*(log(8.1-x)-log(x+8.1));
y3D = C3D*(1./(8.1-x)-1./(x+8.1));
x_exp = [-7,-5,-3,-1,1,3,5]; %vector for all x
y_exp = [-1.5,-1,-0.5,0,0.5,1,1.5]; %vector for all y
hold on;

plot(x_exp, y_exp, 'ko');
plot(x, y2D, 'r');
plot(x, y3D, 'b');
grid on;
title('Potential vs. position');
legend('exp','2D', '3D');
xlim([-8.1 8.1]);
ylim([-1.75 1.75]);
xlabel('Position(cm)');
ylabel('Potential(V)');
set(gca,'XTick',-8:8);
hold off;
```



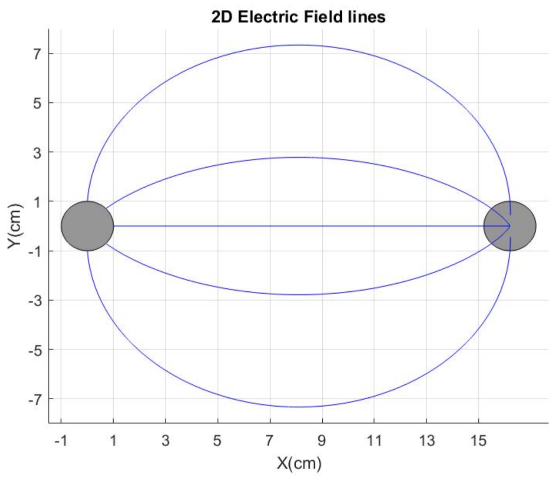
Exp 1: potentials along the midline

```
C2D = -0.54452;
hold on;
for i = -1.2:0.3:1.2
    alpha = exp(2*i/C2D);
    f = @(x,y) (x+8.1).^2-alpha*(8.1-x).^2+y.^2-alpha*y.^2;
    fimplicit(f, [-10 10 -8 8], 'b-');
end
grid on;
theta = linspace(0,2*pi*1000);
fill(-8.1 + cos(theta), sin(theta), [150 150 150]/255);
fill(8.1 + cos(theta), sin(theta), [150 150 150]/255);
axis normal;
title('2D equipotential lines');
legend('equipotential lines');
xlim([-10 10]);
ylim([-8 8]);
xlabel('X(cm)');
ylabel('Y(cm)');
set(gca, 'XTick', -10:10, 'YTick', -8:8);
hold off;
```



Exp2: 2D equipotential lines

```
x = linspace(-1,17.2,1000); y = linspace(-7.5,7.5,1000);
[X,Y] = meshgrid(x,y);
R1 = sqrt(X.^2+Y.^2); R2 = sqrt((X-16.2).^2+Y.^2);
U = -log(R1)+log(R2);
[Ex,Ey] = gradient(-U);
phi = (-90:45:90)*pi/180;
xx = cos(phi); yy = sin(phi);

hold on;
theta = linspace(0,2*pi,1000);
fill(cos(theta),sin(theta),[150 150 150]/255);
fill(16.2+cos(theta),sin(theta),[150 150 150]/255);
streamline(X,Y,Ex,Ey,xx,yy,[0.1,12500]);
grid on;
xlim([-1.5 17.7]); ylim([-8 8]);
xlabel('X(cm)'); ylabel('Y(cm)');
axis normal;
title('2D Electric Field lines');
set(gca,'Fontsize', 10,'XTick', -1:2:16.2, 'YTick', -7:2:7);
hold off;
```



Exp3: 2D Electric field lines